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PORTFOLIO ANALYSIS,

a Special Issue

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William F. Sharpe*

Someone has said that maternity is a matter of fact, while paternity is, at best, a matter of opinion. Concerning the field to which this issue of the Journal is devoted, however, there is little difference of opinion -- Harry Markowitz is almost universally considered to be the father of portfolio analysis. The essence of the theory appeared in his 1952 article[1], although the full exposition was not available until several years later.[2]

The Background

Formally, portfolio analysis theory deals with the selection of a set of investments. Markowitz suggested that the investor should be interested in the probability distribution of his future wealth, and that the selection process should deal, at the very least, with some measure of dispersion of this distribution as well as with its central tendency. For purposes of computation, the standard deviation (or variance) is an attractive candidate for the former, and the expected value for the latter.

Assuming these measures are sufficient, the investor's problem can usefully be separated into three tasks. First, he must obtain estimates of future outcomes for individual securities; this is the task of security analysis. However, these should not be simply point-

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[1]Harry M. Markowitz, "Portfolio Selection," The Journal of Finance, March, 1952, pp. 77 - 91.

[2]_____, Portfolio Selection: Efficient Diversification of Investments (New York: John Wiley and Sons, Inc., 1959).

estimates; explicit account should be taken of the risk of each security and, even more important, the extent to which its outcome is likely to be related to those of other securities. Given a suitable set of such estimates, the investor needs to determine the set of efficient portfolios. A portfolio is defined as efficient if (and only if) it offers a higher overall expected return than any other portfolio with comparable risk (i.e. standard deviation of return). This second task, portfolio analysis, cannot be performed by simply enumerating possible portfolios. One of Markowitz' major contributions was an algorithm specifically designed to perform this task -- formally, a problem in parametric quadratic programming. The final problem facing the investor is portfolio selection -- given the set of efficient portfolios, which one should he hold? This may depend entirely on his feelings towards risk vis-a-vis expected return. On the other hand, market mechanisms may play a part in the decision.

It seems almost incredible now, but until Markowitz suggested this approach to portfolio analysis, no rigorous basis existed to justify diversification; arguments were ad hoc at best. Moreover, the concept of risk, although a major element of finance (as opposed to economic) literature, was rarely even defined rigorously, let alone treated analytically. Markowitz both specified a framework that met these issues directly and provided a workable algorithm for employing that framework for practical problems. The intent was primarily normative, however. Markowitz did not suggest that people actually do select portfolios as if they had solved complicated problems in parametric quadratic programming; rather he suggested that they should select them in this manner. It is also important to note that he did not suggest a preferred technique for security analysis nor the appropriate method for portfolio selection. His interest lay in proposing a general structure for the overall process and providing an algorithm for performing the task of portfolio analysis.

As might have been expected, there was little evidence during the first few years after its initial publication that Markowitz' contribution would have a major impact. Now, however, it is becoming apparent that this was only the top of the iceberg. Articles on portfolio

analysis and related subjects are becoming commonplace in such journals as Management Science, The Journal of Business, The Journal of Finance and, of course, this Journal. And the number of doctoral candidates in finance writing dissertations in the area gives the appearance of approaching infinity. In short, academic interest in the theory is intense. Needless to say, applications to real-world situations have lagged behind (as they undoubtedly should). But the first hesitant steps are now being taken by some of the more venturesome individuals and institutions. One can easily predict that these efforts will be greatly expanded in the future.

It is difficult to briefly describe the many paths down which portfolio analysis theory has led investigators, but some attempt may prove worthwhile. To keep the discussion brief, I will omit many names and all references; the interested reader will find most of the key articles referenced in the papers that follow.

An extremely important application and extension of the theory was made in 1958 by Tobin, who cast it in a positive role, suggesting that the theory offered a plausible explanation for liquidity preference. He also showed that the portfolio selection problem could be partly solved by considering the market for risk-free funds. Given the unpalatable, but perhaps acceptable, assumption that an investor can borrow or lend money at a single "pure" interest rate, there exists a preferred efficient combination of risky investments; the investor need only lever himself up or down by borrowing or lending to obtain the position consistent with his particular attitude towards risk vis-a-vis expected return. This "Tobin effect" has been proven to have major implications for both positive and normative applications of the theory.

A number of investigators have considered the question of the utility of the Markowitz procedure in a normative role. Given some method of security analysis and some rule for portfolio selection, does the Markowitz approach provide portfolios that outperform those selected in other ways? This is, of course, an empirical question; and all the usual problems associated with such questions arise. What is the appropriate test of performance? Are the methods reproducible

in the future? And, if the test shows the Markowitz technique to be inferior, doesn't this simply suggest that the security analysis method utilized was poor? The majority of the tests performed to date have utilized past performance to derive (probabilistic) estimates of future performance. The alternative is usually a randomly-selected portfolio or the actual performance of a mutual fund. Surprisingly, even under these conditions, the Markowitz approach appears to stand up well. The evidence, although limited, is thus comforting to the theory's proponents.

A related line of investigation has dealt with simplified versions of the Markowitz approach. The objective is to economize on the number of inputs required for portfolio analysis (and thus necessary as outputs from the security analysis process) as well as the computer time required to perform the portfolio analysis. Some of the work has been theoretical, dealing primarily with efficient solution algorithms. But more recent investigations have been primarily empirical. Granted a simplified method should cost less to implement; is the (presumed) loss in performance small enough to make its selection worthwhile? Two types of tests have been employed. It is possible that the simplified method will produce portfolios virtually identical to those produced by a full analysis. If so, the test is over and the victory complete. If the portfolios do differ however, the argument is not settled; a second test must be employed. Do the portfolios selected by the simple method perform as well in the future as those selected with a full analysis (or at least well enough to make it worthwhile to reduce the cost of implementation accordingly)? Of course, hundreds of simplified methods can be imagined. Some have been tested already -- many more will be in the future.

The Markowitz approach as a normative theory of investment selection is increasingly being investigated empirically rather than theoretically. However, much theoretical work is being done along related lines. A number of investigators have rejected one or more of the bases on which the Markowitz theory is constructed. Many have argued that the mean and variance of future wealth are not sufficient parameters for decision making. Others have suggested that the probability distribution of a

single security's rate of return is best approximated by a distribution with an infinite variance; under these conditions (a fortiori), a calculus based on variance has no meaning. Surprisingly, however, many of the conclusions of the Markowitz approach (especially those obtained with some of the simplified versions) hold even under these conditions. Other investigators have suggested that the investor should attempt to maximize expected wealth subject to constraints on the probability of losses of specified types. Although such chance-constrained programming approaches are formally competitive to the Markowitz theory, in spirit they have much in common with an extension of the latter suggested by Baumol.

Yet another school prefers to view the investor as considering his position under various states of the world, without simply assigning probabilities to such states and then proceeding along the lines of the Markowitz theory. It is far too early to suggest the final outcome, but it may well be that this state-preference theory will eventually coexist peacefully with the Markowitz theory as do the quantum and wave theories of light -- i.e. each will be considered applicable for a specified range of problems.

In finance and economics, models developed for normative purposes are often used for positive theories, and vice-versa. Portfolio analysis is no exception. Tobin's application has already been mentioned; much additional work is being done. As in traditional economic theory, it is assumed that individual decision makers act as if they were investigating alternatives, selecting efficient strategies, and trading on a relatively efficient market to the extent desirable, given their preferences. The difference is that risk is included explicitly. Given these assumptions, what kind of equilibrium conditions are to be expected? And, most important, how will key elements of the economy respond to changes of various types?

Most of the work in the area of positive theory has been both theoretical and concerned with capital (investment) markets per se. Empirical results are only now beginning to appear. By and large, they suggest that the world is reasonably consistent with the implications of the theory. But it is far too early to render any final judgement.

A fascinating line of inquiry along theoretical lines has been

virtually untapped thus far, and it promises to be the more important. One of the major characteristics that distinguishes finance (and, indeed, business administration generally) from traditional economics is its insistence that risk permeates the business world and must be dealt with explicitly. If the Markowitz approach is useful in the context of security selection then it must be useful in almost all areas of finance (and probably most other areas of business administration). The textbooks in business finance need to be rewritten just as badly as do those dealing with investments. Some of the major theoretical contributions of the future will come, I predict, from those applying and extending the Markowitz approach to cover this broader field. Meanwhile, the continuing decrease in the cost of computer time, the continuing increase in the size of high-speed computer memory, the increasing availability of equipment allowing interaction between an investigator and his data, and the availability of large amounts of security data in machine-sensible form will lead to extensive empirical investigations in the areas of security analysis and portfolio analysis.

This Issue

The papers in this issue of the Journal exemplify the range of work that has been done in the past and make important contributions towards the resolution of a number of outstanding issues. Wallingford's article provides a description of the Markowitz approach, some of the suggested simplifications and much of the empirical work that has been performed to date. The primary concern of the paper is, however, the desirability of alternative simplifications of the Markowitz approach.

Wallingford's preliminary evidence tends to support a hypothesis put forth by Cohen and Pogue -- when a mix of security types (e.g. common and preferred stock) is under consideration, a more complicated model should outperform a less complicated one. As Wallingford indicates, the results of this study provide at best presumptive evidence. The most relevant test (performance in future periods) was not performed. Moreover, his experiments differed in important respects from those of Cohen and Pogue. Perhaps the most intriguing question

in this connection concerns the imposition of upper boundson holdings. Such bounds are typically added for practical applications, either to reflect legal constraints or to force extensive diversification. However, their imposition tends to dampen differences in results obtained using alternative approaches. This may well explain many of the differences among the results of Cohen and Pogue, Wallingford and (also in this issue), Renshaw. The relevant question is, however, largely unanswered. In terms of future performance, what is the preferred method, where "method" refers to the underlying model and the imposition or lack of imposition of upper bounds?

The valuable paper by Samuelson is in the tradition of dissent from the Markowitz approach per se. Samuelson is concerned with the more general class of investments whose returns follow Pareto-Lévy distributions (of which the normal distribution is simply a special case). In general, the concept of variance is inapplicable in this case, but an alternative measure of dispersion is available. Assuming independence of investments, Samuelson posits an approach similar in many respects to that of Markowitz; there is even an analogue to the Tobin effect. Once interdependence of investments is acknowledged, however, the problem becomes considerably more complicated, although an approach similar to some of the simplified versions of the Markowitz method appears to be feasible. This result (similar to that shown earlier by Fama), rescues major elements of the Markowitz approach, even in the hostile environment of a world in which variances can best be approximated by infinity.

The third paper, by Renshaw, includes discussions of additional material from the field (e.g. Latanes terminal-wealth criterion, Fisher's formula relating the geometric mean to the arithmetic mean and variance, and Baumol's criterion for a cutoff point along the efficient frontier), but the central concern is with simple alternatives to the Markowitz approach. Renshaw's alternatives are very much in the spirit of portfolio analysis theory, but cannot be derived explicitly from it. The most interesting suggestion concerns the use of a simple criterion for ranking securities, one which I like to term the reward-to-variability ratio. The numerator is the difference between an historical rate of

return and the pure interest rate; it represents the reward for bearing risk. The denominator measures the risk actually borne (via the standard deviation of rate of return). The ratio is thus the reward per unit of risk. The desirability of this ratio as a measure of ex post portfolio performance follows directly from the Tobin effect, although it carries considerable intuitive appeal as well. But theoretically, it is not particularly relevant for evaluating individual securities. Renshaw suggests that empirical evidence may show that it is useful in this latter role after all, and offers some presumptive evidence. Needless to say, much more must be done before this suggestion can be accepted. We need to determine the extent to which such simple procedures really do lower cost; after all, the cost of computation has been falling dramatically and is expected to continue to do so. More important, we need to compare the future performance of portfolios selected using Renshaw's methods with that of portfolios selected using Markowitz techniques. But Renshaw's study suggests that such tests may yield a number of previously unexpected results.

The interesting paper by Hofflander and Duvall, is in the tradition of normative models of insurance firms, with extensions along the lines of the Markowitz approach. The cost of a policy (the counterpart to the rate of return on an investment) is assumed to be independent of that of another policy within the same line of insurance. But the costs of different lines are assumed to be correlated. The viewpoint of the model is that of the manager of the company in question. No account is taken of the possibility that the company's owners may view decisions differently since they can (and presumably do) hold securities of other firms as well. Here again, the view is primarily traditional.

The paper by Michaelsen and Goshay departs significantly from tradition. On the empirical level, it too is concerned with insurance firms. But it is positive in outlook. Moreover, the authors come to grips with the fact that owners are likely to hold portfolios including many securities. Portfolio analysis theory, by and large, has been unable to say much about the policies of financial intermediaries, let alone corporations in general. If investors can alter their portfolios, and in particular, if they can substitute home-made leverage for

corporation-made leverage, what attitude should a manager take towards risk? Michaelsen and Goshay suggest that the direction of cause and effect may be opposite to that traditionally assumed, with the manager selecting the corporation's attitude toward risk and investors adjusting their holdings accordingly. This view, if correct, has widespread applicability. Michaelsen and Goshay also present empirical results that further lend considerable support to the hypothesis. It seems appropriate to conclude this issue of the Journal with the Michaelsen Goshay paper, since it points the way for the assault on the traditional theory of business finance. In my view, this assault is long overdue, and this paper should prove to be of major importance in the field.

This issue of the Journal should convince even the most reluctant of the traditionalists that portfolio analysis theory is not a transitory phenomenon. If it is not now in the mainstream of finance, it soon will be.