RISK-AVERSION IN THE STOCK MARKET: SOME EMPIRICAL EVIDENCE

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March 1965

P-3084

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A recent article[1] explored the conditions for equilibrium in the market for capital assets under the assumptions that investors are risk-averters, have similar (probabilistic) beliefs about the future performance of various assets, and can borrow or lend funds at a common (pure) interest rate. Briefly, the article showed that under such conditions, market prices of capital assets will adjust so that the predicted risk of each efficient portfolio's rate of return is linearly related to its predicted expected rate of return. Letting σ_1 stand for the standard deviation of the subjective probability distribution of the rate of return on an efficient portfolio, and E_i for the expected value of the distribution, the prices of capital assets will adjust until all efficient portfolios conform to the relationship:[2]

$$E_i = p + b\,\sigma_i$$

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[1] W. F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," The Journal of Finance, Vol. XIX, No. 3 (September 1964) pp. 425-442.

[2] By definition, for an inefficient portfolio:

$$E_i < p + b\,\sigma_i.$$

where p is the pure (riskless) interest rate and b(> 0) is the risk-premium. An important corollary concerns the relationship among the predicted rates of return on efficient portfolios: under the assumed conditions they will be perfectly correlated.

Strictly speaking, the implications of this theory cannot practically be tested, since the relationships refer to predictions concerning expected returns from assets and the associated risks. Clearly actual results may diverge considerably from the predictions made by investors at the time they purchase assets. Moreover, investor preferences and investment opportunities presumably change over time. This poses a major problem: if the equilibrium values of p (the price of time) and b (the price of risk) change from year to year, it may be dangerous to use data from several years to estimate their average values. But if the results from several years are not used, how can predicted values of E_i and σ_i be estimated? In many respects the problem is similar to that of measuring demand curves. If any empirical tests are to be performed, rather stringent assumptions must be made.

To provide some evidence concerning the behavior of the capital market, the annual returns from 34 open-end mutual funds[3] during the period from 1954 through 1963 were analyzed in the manner suggested by the theory. The average rate of return[4] for each fund over the ten-year period was used as an estimate of its expected rate of return (E_i). Similarly, the standard deviation[5] of the actual rate of return

[3]The funds selected were those for which annual rates of return were given by Wiesenberger for at least the last twenty years. All data are from Arthur Wiesenberger and Co., Investment Companies, 1962 and 1964 editions.

[4]In all cases, rate of return is based on the sum of dividend payments, capital gains distributions and changes in net asset value.

[5]Derived from an estimate of variance computed by dividing the sums of the squared deviations by (N - 1).

for each fund over the 10-year period was used as an estimate of its predicted risk (σ_i). Lacking any satisfactory test for reasonable diversification, all 34 funds were assumed to have chosen efficient combinations of securities.[6] Needless to say, the measures utilized cannot be expected to perfectly reflect investors' predictions and some mutual funds may hold rather inefficient portfolios; thus the relationship between the values of σ_i and E_i in the sample will be approximate at best. But if the theory is worth any consideration at all, there should be such a relationship, it should be significant, and funds experiencing greater variability should provide greater average returns.

Table 1 shows the results for the 34 funds in the sample; the data are plotted in Figure 1. The predicted relationship is clearly present. The correlation coefficient between σ and E is + .836, highly significant[7] and consistent with the assumption of risk-aversion.

It is not a simple matter to select appropriate estimates for the pure interest rate and the risk-premium from the sample data. The market process by which E_i and σ_i values are made to follow a linear relationship involves changes in the prices of capital assets; the price changes in turn alter the values of both E_i and σ_i.[8]

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[6]The return on a fund is, of course, slightly less than the return on its portfolio, due to the expenses incurred by the fund's management. For the purposes of this analysis, the two are assumed to be equal.

[7]A coefficient of .45 is significant at P = .01; the t-value of the slope of the regression line is 8.61.

[8]Recall that E_i and σ_i refer to the mean and standard deviation of the rate of return.

Table 1

Mutual Fund	Average Annual Return 1954-1963 (E_i)	Standard Deviation of Annual Return 1954-1963 (σ_i)
Affiliated Fund	14.6%	15.3%
American Business Shares	10.0	9.2
Axe-Houghton, Fund A	10.5	13.5
Axe-Houghton, Fund B	12.0	16.3
Axe-Houghton, Stock Fund	11.9	15.6
Boston Fund	12.4	12.1
Broad Street Investing	14.8	16.8
Bullock Fund	15.7	19.3
Commonwealth Investment Company	10.9	13.7
Delaware Fund	14.4	21.4
Dividend Shares	14.4	15.9
Eaton and Howard, Balanced Fund	11.0	11.9
Eaton and Howard, Stock Fund	15.2	19.2
Equity Fund	14.6	18.7
Fidelity Fund	16.4	23.5
Financial Industrial Fund	14.5	23.0
Fundamental Investors	16.0	21.7
Group Securities, Common Stock Fund	15.1	19.1
Group Securities, Fully Administered Fund	11.4	14.1
Incorporated Investors	14.0	25.5
Investment Company of America	17.4	21.8
Investors Mutual	11.3	12.5
Loomis-Sayles Mutual Fund	10.0	10.4
Massachusetts Investors Trust	16.2	20.8
Massachusetts Investors -- Growth Stock	18.6	22.7
National Investors Corporation	18.3	19.9
National Securities -- Income Series	12.4	17.8
New England Fund	10.4	10.2
Putnam Fund of Boston	13.1	16.0
Scudder, Stevens and Clark Balanced Fund	10.7	13.3
Selected American Shares	14.4	19.4
United Funds -- Income Fund	16.1	20.9
Wellington Fund	11.3	12.0
Wisconsin Fund	13.8	16.9

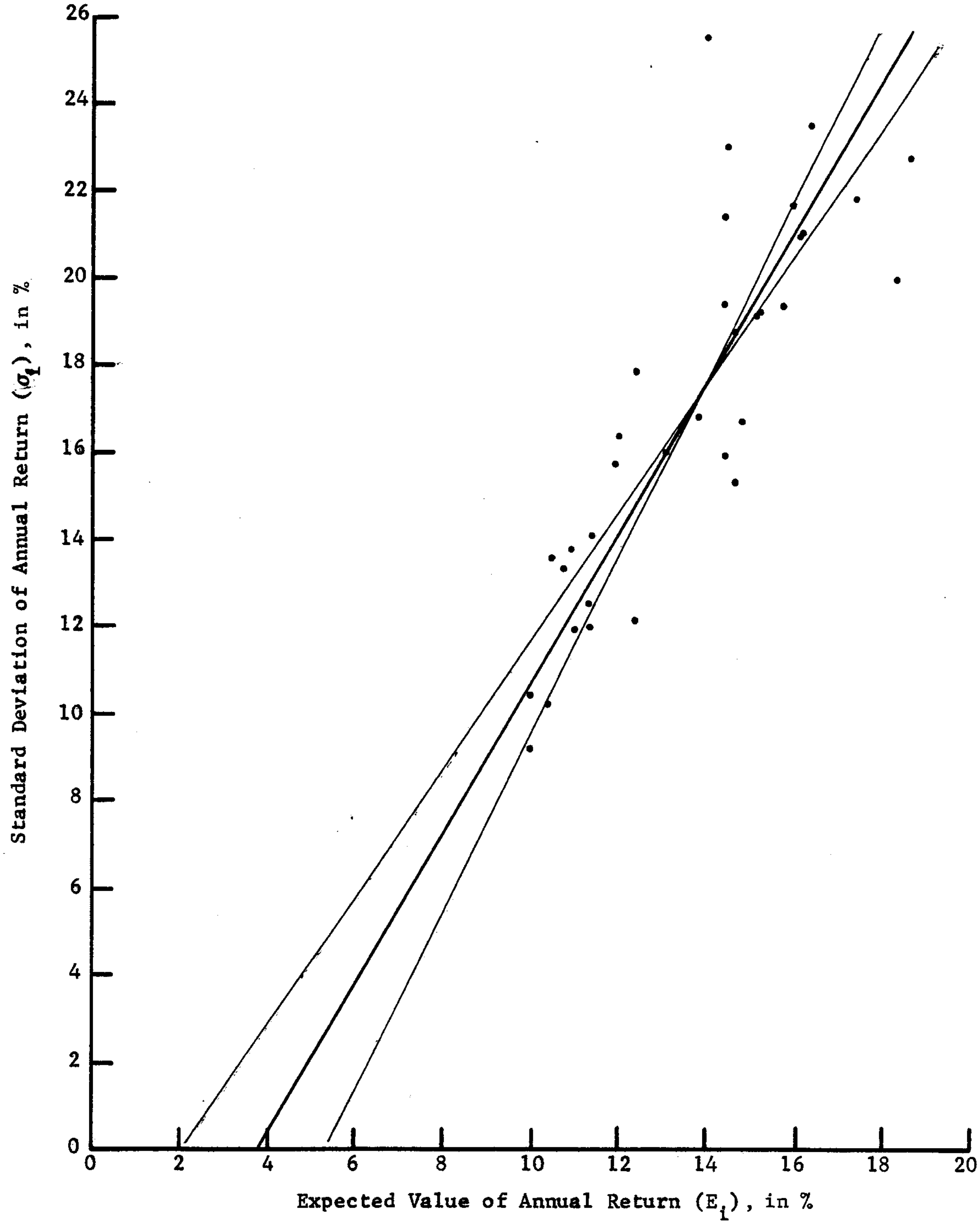
26
24
22
20
18
16
14
12
10
8
6
4
2
0
Standard Deviation of Annual Return (σ_i), in %
0
2
4
6
8
10
12
14
16
18
20
Expected Value of Annual Return (E_i), in %

Figure 1

Since neither can properly be considered an independent variable, the appropriate line for estimating their relationship presumably lies between the one obtained by regressing σ on E and that obtained by regressing E on σ. Figure 1 shows the two regression lines and an intermediate one. The latter is simply the line passing through the intersection of the two regression lines[9] with a risk-premium (b) equal to the average of the values for the regression lines. The estimates of the pure interest rate and the risk-premium corresponding to the three lines are:

	p	b
Regression line: σ on E	2.06	.678
Regression line: E on σ	5.55	.474
Intermediate line	3.81	.576

The values obtained from the intermediate line can be interpreted as follows. During this period investors required (and got) an annual rate of return of approximately 3.8 per cent on riskless assets. To take on risk, they required (and got) an additional .58 per cent of expected return per annum for each 1 per cent of predicted standard deviation of annual return (risk). These values may or may not properly characterize the conditions in the capital market during the period covered by the sample. Even if they do, there is no a priori reason to assume that they will characterize any future period. However, for purposes of prediction these results may be as useful as any others presently available.

[9]This point corresponds to the average values of E_i and σ_i for the funds in the sample. The values are:

$$\bar{E}_i = 13.64\% \text{ per annum}$$

$$\bar{\sigma}_i = 17.07\% \text{ per annum.}$$

Thus far we have shown that higher values of σ_i are associated with higher values of E_i but we have not tested directly the implication of the theory that the relationship is linear. This conclusion is a direct result of the assumption that investors can either borrow or lend at a common pure interest rate. If an investor's funds can be divided between a risky portfolio and an investment at the pure interest rate, any combination of E and σ on the line connecting the two points is attainable. If the investor is able to borrow funds at the same rate of interest, any point on the extension of the line to higher values of E and σ is also attainable. Under these conditions all efficient portfolios must lie along a straight line passing through the point representing the pure interest rate. If the assumptions are violated, of course, this relationship need not hold. Since investors are probably unable to borrow extensively at the pure interest rate in order to purchase risky assets, we should not be too surprised to find that the relationship between σ and E turns upward from a straight line as higher values of E (and σ) are reached.[10]

There is some evidence of such curvature in the sample data. A quadratic regression equation gave a slightly higher correlation coefficient than did the linear regression equation -- (+ .852

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[10] For a discussion of the reasons behind this assertion, see Sharpe, op. cit., pp. 432, 433.

instead of + .836)[11] and the curvature was of the type anticipated. This result could be due to the presence of one or more relatively inefficient portfolios in the region of high risk,[12] but no satisfactory method for testing the validity of this explanation is available. In any event, although the implication of linearity is violated to some extent, it seems reasonable to assert that for purposes of characterizing the general nature of the capital market, the relationship between σ and E for efficient portfolios can safely be assumed to be linear.

If all efficient portfolios lie along a straight line in the (E,σ) plane, their predicted rates of return will be perfectly correlated. Putting it another way, the predicted rate of return of any single portfolio should be linearly related to that of any other we might choose for the comparison. If each portfolio is compared with the same (standard) portfolio, the correlation coefficients should all equal +1. As before, we can only use actual results as surrogates for predicted performance, and thus cannot expect perfect correlations. However, if an appropriate standard is selected, relatively high values should be obtained.

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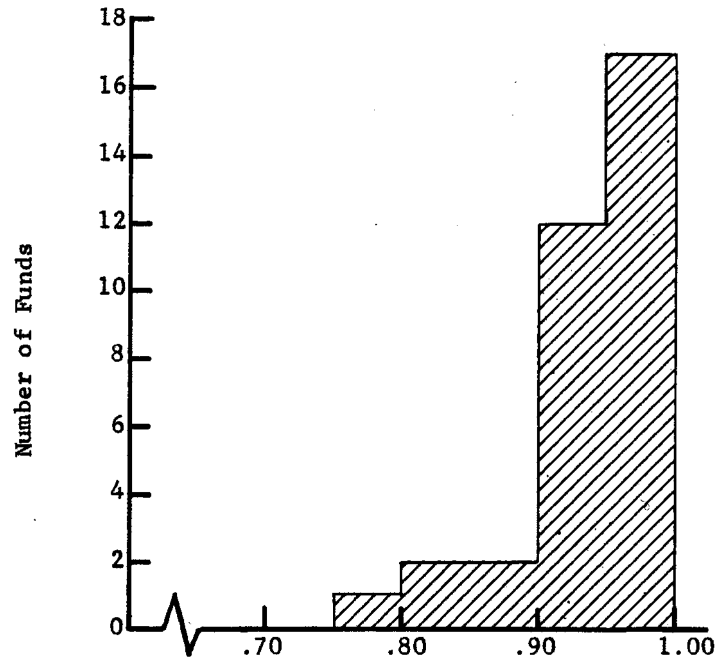
[11]The regression equation was of the form: $E = a + b\sigma + c\sigma^2$. The t-values of the latter two coefficients were :

b : + 2.87
c : - 1.78 .

The relative magnitudes of the two correlation coefficients given in the text overstate slightly the advantage of the quadratic equation since one more degree of freedom was lost in its estimation. A χ^2 test of the normality of the distribution of the residuals from the regression of σ on E shows that the assumption of linearity is significant at p = .30.

[12]The fund with the highest standard deviation may well be guilty of this charge.



Correlation Coefficient: Rate of Return on Fund vs Rate of Return on Dow-Jones Portfolio, 1954-1963

Figure 2

To test this implication of the theory, the actual rates of return over the ten-year period for each fund were correlated with those of the 30-security portfolio used to compute the Dow-Jones Industrial Average. Figure 2 shows the distribution of the correlation coefficients for the 34 funds. Needless to say, none was +1, and the sample size in each case was only ten. However, although no satisfactory test for consistency can be employed, the results certainly appear to be in substantial agreement with the theory.

This concludes the evidence. Although fragmentary, and not even particularly novel,[13] the data do lend considerable support to the theory tested. The issues surrounding the behavior of the capital market are, however, far from settled. It is entirely possible that data obtained from different segments of the market and different time periods may not conform to the relationships predicted by the theory and found here. Hopefully, more detailed and extensive investigations will be made in the future; the importance of the subject certainly warrants the effort.

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[13] A similar analysis can be found in W. F. Sharpe, Portfolio Analysis Based on a Simplified Model of the Relationships Among Securities, PhD. Dissertation, UCLA, June 1961. A sample of 23 mutual funds showed similar relationships in two separate periods: 1940-1951 and 1951-1959. In the study leading to his recent article, "How to Rate Management of Investment Funds," The Harvard Business Review, January/February 1965, Vol. 43, No. 1, pp. 63-75, Jack L. Treynor apparently obtained results similar to those given here (although they are not stated explicitly in the article). The tests described here are sufficiently obvious that it seems reasonable to assume that they have been performed elsewhere as well, although no results appear to have been published.