Individual versus Institutional Investing

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This paper first describes the analytic approach that Markowitz used in developing his portfolio theory. Developing a game-of-life simulation is a parallel approach for modelling individual financial management. To develop a realistic simulator will require deciding what goals are essential to the family planning process, formulating optimizable subproblems, using technology to interpret and record decisions, and developing decision rules which prove robust in the model and can be implemented in practice.

Professor Mandell, editor of *Financial Services Review*, invited me to contribute an article related to financial research for the individual for the first issue of this journal. Since the subject is not my specialty, it was uncharacteristically risky of me to have accepted the invitation. But an evening of reflection convinced me that there were clear differences in the central features of investment for institutions and investment for individuals, that these differences suggest differences in desirable research methodology, and that a note on these differences may be of value.

As I thought about the subject further, on subsequent days, I found myself of two minds. On the one hand, surely financial decisions for the individual should be considered as part of the "game as a whole" which the individual plays out—"game" in the sense of von Neumann and Morgenstern (1944). Even reducing this game to its essentials, it has characteristics of situations for which simulation methods have proved to be the superior tool in practice. On the other hand, there are approximations to the individual's financial situation which seem good enough, and would allow us to solve analytically for optimal action.

Neither train of thought succeeded in defeating the other. Below, I present both views: the first in a section called "Thesis," and the second under "Antithesis," and attempt some reconciliation in a final section, "Synthesis."

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THESIS

In Markowitz (1952) I conjecture that "Perhaps—for a great variety of investing institutions which consider yield to be a good thing; risk, a bad thing; gambling to be avoided—E, V efficiency is reasonable as a working hypothesis and a working maxim." The "investing institution" which I had most in mind when developing portfolio theory for my dissertation was the open-end investment company or "mutual fund." This familiarity was not from first hand experience (I was a student and son of a grocer) but from Wiesenberger & Company's *Investment Companies* (1944-   ). It was plausible to assume for the mutual fund that its objective is to obtain a "good" probability distribution of year-to-year (or quarter-to-quarter) percent increase in its net asset value. In addition, I argued for mean and variance as criteria in judging "goodness." For the present discussion, the choice of mean-variance criteria is not the crux; rather it is the formulation of the problem as that of selecting a portfolio to achieve a good probability distribution of a single random variable: the return on the portfolio as a whole. This formulation turned out to be widely acceptable in practice as well as tractable analytically.

In the 1950s, I participated in attempts to develop advanced, but practical, methods for assisting manufacturing planning, particularly assisting equipment selection and production scheduling for job shops. We considered optimization techniques first, such as linear and dynamic programming, but found that too much reality had to be ignored to allow these techniques to be applied. Simulation techniques seemed promising, and were developed for real decision problems with real job shops. Experience confirmed the value of simulation analysis, but showed that programming the model was a bottleneck. This, and similar experiences in other application areas, stimulated development of the simulation programming languages of the early 1960s.

In the meantime, attempts continued to apply analytic techniques to shop scheduling problems. A recent survey (Lawler, Lenstra, Rinnooy Kan, and Shmoys, 1989) reports that some flow shop problems have been solved; others have been shown to be NP-hard (i.e., as hard to solve as the traveling salesman problem); but results for job shops are meager, leading the authors to end with a quote from Coffman, Hofri, and Weiss (1989), "there is a great need for new mathematical techniques useful for simplifying the derivation of results." In the meantime, simulation analysis is increasingly used in practice.

The difference between the investment company situation and that of a job shop is the number of state variables that need to be considered in a practical problem. For the investment company, it is plausible to assume that assets are liquid, therefore the state of the portfolio can be described by its total value. For the shop, its state description includes the contents of all its queues.

To judge whether the problem of financial planning for the individual is amenable to analytic solution, let us sketch what a "game-of-life" model might

entail. We seek a model with sufficient realism as to be a guide to practice. For example, some economic theories find it convenient to assume that the individual is immortal, or that death is a Poisson process independent of the age of the individual. For actual financial planning, however, aging and mortality are salient facts that must be included in the model. On the other hand, many details of life which are important to the individual may be ignored for financial planning. For example, the model should include the probability of an accident or disease which will keep the individual from work for an extended period, the probability distribution of time to recover or die, costs of treatment and probability of relapse, since these possibilities are major factors in financial planning; but medical details are not required.

Since time and uncertainty are at the heart of the problem, I will sketch the model as if it were a simulation. This is for the purpose of model description, and does not itself preclude the possibility that the model could be solved analytically. The description will use the SIMSCRIPT worldview (Kiviat, Villanueva, and Markowitz, 1983). This says that, as of any instant in time, the model represents *entities* of various *entity types*; a given entity is characterized by the *values* of its *attributes*; also, it may own *sets* to which other entities belong, and belong to sets which other entities own. This status description changes at points in time called *events*.[1] One event may cause one or more subsequent events to occur after fixed or random time delays.

The essentials of the game-of-life is probably different for (a) the very wealthy, (b) the class of homeless that used to be called vagrants, and (c) most of my friends and relatives. I have the latter in mind as I sketch the model.

Among the types of entities of the model, we must distinguish between the *individual* (i.e., human person) and the (nuclear) *family*. Often, at some stage this family will "own" a set of individuals whose roles are husband, wife, children and perhaps residing elder. Frequently, in the course of events, the residing elder (if any) dies or is placed in a nursing facility; the children leave home to set up their own nuclear families; the original family (the subject of the model) then consists of husband and wife. When one dies the subject family consists of the survivor only. When the latter dies, the assets of the subject family are distributed to heirs, and the game-of-life is over for the subject family.

In the simplest case, assets may be thought of as belonging to the family rather than the individual, to be used by husband and wife and (at their discretion) by the children until, at the last, it is used to support the survivor and then distributed. It may be sufficient to characterize financial assets as the total value of the family's holdings in stocks, bonds, cash items and real estate [other than the family's home(s)]. Perhaps, upon further reflection, it may prove essential to disaggregate these items according to the maturity of the bonds, their tax exempt status, and the unrealized capital gains and losses of various assets. [Problem: must we distinguish many individual stocks in order to characterize available capital gains and losses for tax calculations?] Perhaps

bonds and stocks may be treated as instantaneously marketable, perhaps with a small commission, but real estate requires greater (random) time and cost to sell.

Among other assets, the family may "own" [in the SIMSCRIPT sense, i.e., have associated with it] one or more residences. The residence may be owned (in the usual sense) or rented. If owned, the residence is characterized by original cost and a current market value; whether owned or rented, the residence has a value of owned furnishings. A home and its furnishings are clearly an illiquid asset, not only because of the time and cost to sell, but also that to move, and the mismatch between furniture needs of the old and new residences.

Attributes of individuals include those needed to characterize health, the employment or employability of husband and wife, and the educational objectives of each child. The assets of a residing elder can be characterized by associating with this entity his or her own nuclear family entity.

Events which change status include periodic events such as receiving a salary check, having a birthday, or the time when an income tax payment is due; and randomly occurring events such as becoming sick, becoming well, finding a job, losing a job, financing a house to buy, finding a buyer for a house to be sold. Changes in price levels, interest rates, and stock and real estate values could be computed periodically; e.g., increments in price levels and interest rates could be drawn from a joint distribution, then the change in real estate values could be computed as a function of the former increments and other random variables.

The simulated family must make decisions at various points in time, such as the level of (say) this week's nondurable consumption, transfers from cash to other liquid assets, the decision to search for and then buy a new house, and the decision of one of its members to retire. The simulated family makes these decisions according to decision rules. A major purpose of the model is to evaluate alternate decision rules.

The above is a partial sketch of a game-of-life model, rather than detailed specifications for one. The model should also include, as essential to evaluating family investment practice in fact, such things as IRAs, Keoghs, social security payments (or the individual's status with respect to future social security payments), status with respect to pension plans, various kinds of insurance, their costs and the kinds of events they insure against (e.g., house fire, car accident).

The model sketched above is, in certain ways, akin to the worksheets published as guides to families; see, e.g., *The Wall Street Journal* (1989). The model differs from the worksheet in that the model allows for many of life's random events—many more such events than one could take into account by filling out alternate, contingent worksheets. Since future status is random, the simulated family must follow adaptive decision rules rather than a single plan as expressed on a worksheet. As noted above, a major function of the model is to evaluate these decision rules.

This sketch of a game-of-life should suffice to convince one that the game is complex; most likely beyond analytic techniques. In contrast, using a good simulation language, it would not be difficult to program as a simulation model *once the specs of the model are decided*. It is unlikely that there will be general agreement as to what should be included in a game-of-life simulator, or how its output should be scored. Therefore it may be expected that there will be more than one game-of-life simulator; and it may be hoped that their respective assumptions will be clearly documented. The various simulators will allow us to see whether rules of behavior which work well in one model will prove robust when tried in alternate models. If so, this will encourage us to recommend them in practice.

In sum, I encourage readers with requisite skills to try building and using realistic game-of-life simulators; and editors to look kindly on the publication of their results.

ANTITHESIS

The problem with simulation analysis is that it is not very good at finding near optimum decision rules. It takes many runs of the model to estimate the excellence of a given set of rules. Since the rules we seek may be adaptive—i.e., may recommend different allocations of resources under different circumstances, and "circumstances" admit to countless variations—it will not be feasible to search for optimum decision rules.

At various points in time in the game-of-life there are requirements for allocating resources among assets. If some of these can be formulated, at least approximately, as portfolio selection problems—where the problem is to get a good distribution of return on the allocation as a whole—then an optimum solution can be found for the approximate problem. If the approximation is a good one then, by definition of good approximation, the exact solution to the approximate problem will be part of a good solution for the more complex game.

For example, consider a family with a house, children a few years from college age, life insurance policies in place based on a separate calculation, which faces the question of whether to shift resources among asset classes such as equities, long term tax exempt bonds, short term tax exempt bonds, etc. Leaving aside, for the moment, the question of unrealized capital gains in the existing portfolio, and assuming that this family does not trade often enough to run up sizable brokerage commissions, then it is plausible to pretend that these assets are perfectly liquid, therefore the value of the portfolio at anytime is the sum of the market value of its constituents, and that the objective in choosing a portfolio of these assets is to get a good probability distribution of return (capital gain plus interest and dividends) for some period of analysis.

Whether the "goodness" of a probability distribution is to be measured by a utility function or by a mean-variance analysis, we must answer questions such as:

(1) How do we measure return? Clearly, the family wants return after taxes. First, if the family realizes capital gain by shifting out of an asset that has an unrealized gain at the beginning of the period, then the tax on this gain must be subtracted from holding period return. Second, if an asset produces income during the period, we must subtract the tax on this income from its return. Third, if an asset has a capital gain during the period then its value to the family is somewhere between its market value and the latter minus the tax if the gain is realized. For simplicity perhaps it is satisfactory to average these two values. Finally, it seems appropriate for the family to seek a good distribution of real rather than nominal return. This raises no problem for the optimizer. (In particular, see Markowitz 1987, chapter 1], concerning the treatment of real returns in a mean-variance analysis.)

(2) What constraints limit portfolio choice? Constraints should consist of those which are imposed by government agencies and brokerage houses on individuals, e.g., limited borrowing and short sales, plus perhaps self imposed constraints such as upper bounds on asset classes which are in fact less liquid than others.

(3) What period of analysis should be used? Do what we always do— pick one.

Admittedly, approximations (and guesses) must be made, but they can be made plausibly. Then the optimum solution can be found to the approximate model. If the approximation is satisfactory, this exact solution to the approximate problem should be part of a good solution for the real problem.

SYNTHESIS

The proposed optimization analysis is only an approximation. A realistic simulator could be used to test decision rules based on optimizing a simplified model as compared to rule-of-thumb decision rules. Also, it is not always clear how the approximation is to be made; e.g., what time period to use for the analysis, how the family should pick a portfolio from the mean-variance frontier, how to treat unrealized capital gains, whether it is sufficient to consider nominal returns or essential to consider real returns, and the like. The simulator could be used to evaluate such alternate methods of formulating the portfolio selection problem within the game-of-life model. Also, a number of investigators

have evaluated the ability of a well chosen point from the mean-variance frontier to approximately maximize the expected value of a single period utility function.[2] Most have concluded that it does quite well for "reasonable" utility functions. This question could be re-examined within the framework of a game-of-life simulation analysis.

The exercise of building a realistic game-of-life simulator—deciding what is essential to the family planning process and incorporating it into a simulator without the severe constraint of producing an analytically tractable model— should be highly educational, especially to the model builders. So should the process of formulating optimizable subproblems and evaluating these within the simulator. Another challenge is to use modern computer technology to help understand and remember what has been done. I have in mind here the use of simulation/animation to display the workings of the simulated world (see CACI, 1988) and the use of some kind of database to allow one to browse the inputs and outputs of prior runs. Finally there is the process of deciding how the decision rules which prove robust in the simulated worlds can be explained and implemented in practice.

I admit that this all seems a lot harder than formulating a highly simplified model that can be solved analytically. But I believe it has more chance of producing credible decision rules for practice—just as simulation analysis continues to produce credible policy recommendations for manufacturing, while analytic methods are not yet available for most sufficiently realistic models in the latter area.

Obviously, results of realistic game-of-life simulators will not be ready for the next issue of this journal. In the short and the long run, we should expect that the *Financial Services Review* will publish research with various approaches to various aspects of its topic area. Such pluralism is desirable in research, as it is in politics and the marketplace.

NOTES

1.   In programming, it is often convenient to bundle events together into *processes*; but for the present discussion it is more convenient to describe events.
2.   Markowitz (1959); Young and Trent (1969); Levy and Markowitz (1979); Dexter, Yu, and Ziemba (1980); Pulley (1981); Pulley (1983); Levy and Markowitz (1984); Reid and Tew (1986); Simaam (1987), Grauer (1986); and Tew and Reid (1987).

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